# Stat 401, section 9.3 Analysis of Paired Data

notes by Tim Pilachowski

In section 9.3, we're still comparing two groups, but now, instead of looking at independent samples, we'll be considering "paired data". Two items/units/subjects in the analysis are matched so that they have characteristics which are as similar as possible.

The usual scenarios are:
 (a) same population: same sample group tested before and after some treatment
 (b) same or different populations: two samples constructed so that each unit in one sample shares important characteristics with a unit in the second sample.

Both Example A and Example B in this Lecture are the second scenario (b).

Basic assumptions:
The data consist of $n$ independently selected pairs $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$, with $E(X_i) = \mu_1$ and $E(Y_i) = \mu_2$.
Define $D_1 = X_1 - Y_1$, $D_2 = X_2 - Y_2$, … , $D_n = X_n - Y_n$.
The $D_i$'s are normally distributed with mean $\mu_D$ and variance $(\sigma_D)^2$. (In most cases, the $D_i$'s are normally distributed because the $X_i$'s and $Y_i$'s are themselves normally distributed.)

As with sections 9.1 and 9.2, we'll be considering the parameter $\mu_1 - \mu_2$. But in the case of paired data, since the pairs are independent of one another, we have $\mu_D = E(X - Y) = E(X) - E(Y) = \mu_1 - \mu_2$. Since we are assuming that the $D_i$'s are normally distributed with mean $\mu_D$, we can construct confidence intervals and conduct hypothesis tests using one-sample methods.

In reality, and in text exercises, the analysis of matched pairs will almost always be a small sample rather than a large sample case, so we'll use the *t*-distribution with $\nu = n - 1$ degrees of freedom to determine the critical value for a confidence interval (section 7.3) and the *P*-value for a hypothesis test (section 8.2b).

Example A – confidence interval. Two groups of 10 students each are matched for gender and for score on a pretest given at the beginning of a study. Group A is taught a skill using a new method, and group B is taught using traditional methods. (The samples have been constructed so that each Group A student has a matching Group B student.) There is evidence that test scores are normally distributed. Using results from a posttest, construct a 95% confidence interval. *answer*: (– 0.578, 3.178)

| Group | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 95 | 86 | 95 | 79 | 92 | 86 | 92 | 82 | 83 | 77 |
| B | 90 | 84 | 97 | 78 | 90 | 85 | 94 | 78 | 85 | 73 |

Side note: Comparing Group A's pretest scores with their posttest scores, and asking "Did the students learn anything?" would also be a paired data case. See "scenario (a)" above.

Example A – hypothesis test. Two groups of 10 students each are matched for gender and for score on a pretest given at the beginning of a study. Group A is taught a skill using a new method, and group B is taught using traditional methods. (The samples have been constructed so that each Group A student has a matching Group B student.) There is evidence that test scores are normally distributed. Using results from a posttest, conduct a test of the hypothesis that the new method produces higher scores than the traditional method ($\alpha = 0.05$).
*answer*: fail to reject $H_0$

| Group | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 95 | 86 | 95 | 79 | 92 | 86 | 92 | 82 | 83 | 77 |
| B | 90 | 84 | 97 | 78 | 90 | 85 | 94 | 78 | 85 | 73 |

Example B. The Federal Trade Commission tests tires to rate them for wear. Brand A is put on one wheel and brand B on another wheel of the same car, matching front to front and rear to rear. Data are amounts of tread remaining, measured in thousandths of an inch. Do brand A tires have less tread left than brand B tires? Conduct a hypothesis test ($\alpha = 0.05$) and state your conclusion. (Assume that distribution of tire tread wear is normal.) *answer*: reject $H_0$

| Brand | | | | | | |
|---|---|---|---|---|---|---|
| A | 125 | 64 | 98 | 38 | 90 | 106 |
| B | 133 | 65 | 103 | 37 | 102 | 115 |