We often learn elementary school math as a bunch of rules, without really understanding the concepts. Now that you're working on a higher level, a really good grasp of those basic ideas will help form a firm foundation for remembering the things you're encountering now.

If you understand how fractions (rational numbers) work, then working with rational expressions is much easier.

FRACTIONS:
Computation and Operations

Although we use fractions all of the time, they can still seem intimidating. Especially frightening is how the rules for addition and subtraction are different from those for multiplication and division. Rest assured, **it's easier than it looks**! The trick is all in the approach.

*ONE MORE TIME: WHAT **IS** A FRACTION?*

Remember, a fraction is a **part** of something that has been cut up into pieces. The numerator is how many pieces you have; the denominator is how many pieces make up a whole.

MULTIPLICATION AND DIVISION

Multiplication of fractions is essentially taking something cut up into fractions and making the pieces smaller by cutting them up again. "$\frac{1}{2} * \frac{3}{4}$" is the same as "$\frac{1}{2}$ of $\frac{3}{4}$". In pictures:

In words, "$\frac{1}{2}$ of $\frac{3}{4}$" gives us $\frac{3}{8}$ of the pie.
Mathematically we achieve that answer by multiplying

is $\frac{3}{4}$. If we take $\frac{1}{2}$ of $\frac{3}{4}$, , we get , or $\frac{3}{8}$!
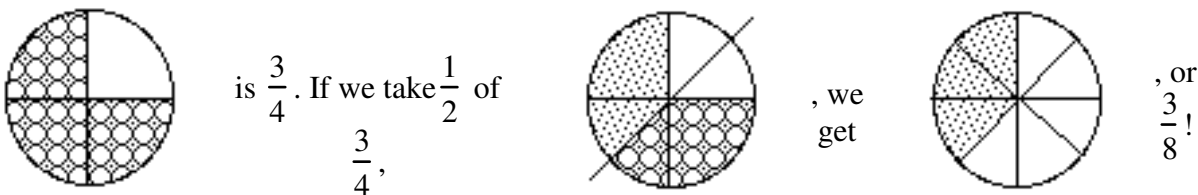
$$\frac{\text{numerator} * \text{numerator}}{\text{denominator} * \text{denominator}} : \quad \frac{1}{2} * \frac{3}{4} = \frac{1 * 3}{2 * 4} = \frac{3}{8}.$$

The same process will work for multiplying **any** fractions! (Notice that you'll need to convert all *mixed numbers* to *improper fractions* before multiplying.)

Division is the opposite (or *inverse*) of multiplication so we can multiply by the i*nverse* (called the **reciprocal**) of the second term, as in:

$$\frac{1}{5} \div \frac{2}{3} \quad = \quad \frac{1}{5} * \frac{3}{2} \quad = \quad \frac{3}{10}.$$

Division in fractions means **multiplying by the reciprocal**. (Once more notice that all mixed numbers must be converted to improper fractions before you do *any* multiplying or dividing!)

RAISING TO HIGHER TERMS AND REDUCING TO LOWER TERMS

The process of changing to higher or lower terms is an extension of multiplication and division. As long as we do the **same thing** to numerator *and* denominator (top and bottom) it's like multiplying or dividing by "1". So, we still have the same amount of "stuff" (equal fractions).

For example, $\dfrac{1}{2} = \dfrac{1}{2} * 1 = \dfrac{1}{2} * \dfrac{2}{2} = \dfrac{2}{4}$ and $\dfrac{6}{9} = \dfrac{6 \div 3}{9 \div 3} = \dfrac{2}{3}$.
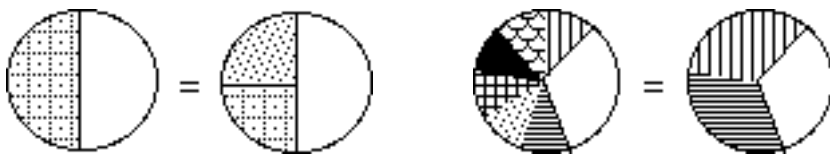
(This procedure, by the way, is an application of Pilachowski's rule #8—Conservation of Stuff. Whenever anything changes, there's always a balance to it!)

We generally **raise to higher terms** to get a common denominator for addition and subtraction (see below). Recall, too, that all answers to fraction problems should be **reduced to lowest terms**.

ADDITION AND SUBTRACTION

In adding and subtracting fractions, there are really only a couple ideas to remember.

1) You can think of fractions like denominate (named or labeled) numbers. Just like,
<div align="center">3 feet + 4 feet = 7 feet,
and 6 pounds + 2 pounds = 8 pounds,</div>

<div align="center">so 3 sevenths + 2 sevenths = 5 sevenths.</div>

<div align="center">that is, $\dfrac{3}{7} + \dfrac{2}{7} = \dfrac{5}{7}$, or</div>

In other words, as long as you have a common denominator (which you can obtain by raising to higher terms), you add fractions by adding the numerators and keeping the denominator!

(We try to get the smallest or **least common denominator** (the **LCD**) because smaller numbers are easier and safer to work with—Pilachowski's rule #6.)

2) Once you have the LCD, all the regular rules of addition (including "carrying") apply.

3) The same goes for subtraction: once you have the LCD, all the regular rules apply. So, in fractions, you can "borrow" when you need to, including borrowing from the whole number part of a mixed number to the fraction part.

FINAL ENCOURAGEMENT

That's all there is to it! As long as you have the basic idea of what's going on, you'll find that working with fractions will be only a quarter as hard as you thought!