We use abbreviation Ex. 1.2.3 means exercise 3 from section 2 of chapter 1 of Weinberger’s book.

1. (Ex.1.5.2) 

\[
\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \exp(x) \quad \text{for} \quad 0 < x < 1 \quad t > 0 \\
u(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1 \\
\frac{\partial u}{\partial t}(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1 \\
u(0, t) = 0 \quad u(1, t) = 0.
\]

Find \( u(0.5, 1.5) \).

2. (Ex.1.5.3) Find the solution of 

\[
\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \sin \pi x \quad \text{for} \quad 0 < x < 1 \quad t > 0 \\
u(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1 \\
\frac{\partial u}{\partial t}(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1 \\
u(0, t) = 0 \quad u(1, t) = 0.
\]

3. (Ex.1.5.5) 

\[
\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 1 - x \quad \text{for} \quad 0 < x < 1 \quad t > 0 \\
u(x, 0) = x^2(1 - x) \quad \text{for} \quad 0 \leq x \leq 1 \\
\frac{\partial u}{\partial t}(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1 \\
\frac{\partial u}{\partial x}(0, t) = 0 \quad u(1, t) = 0.
\]

Find \( u(0.25, 2) \).
4. (Ex. 1.5.6) Show that if the force function $F$ in formula (5.3), from section 1.5 of the book, is a function of $x$ only, the solution $u$ is a periodic function of $t$ of period $2l/c$. That is

$$u(x, t + 2l/c) = u(x, t).$$

5. (Ex.1.6.1)

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \quad \text{for} \quad 0 < x < 1 \quad t > 0$$

$$u(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1$$

$$\frac{\partial u}{\partial t}(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1$$

$$u(0, t) = \sin^2 t \quad u(1, t) = 0.$$ 

Find $u(0.5, 1.5)$.

6. (Ex.1.6.2)

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \quad \text{for} \quad 0 < x < 1 \quad t > 0$$

$$u(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1$$

$$\frac{\partial u}{\partial t}(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1$$

$$u(0, t) = 0 \quad \frac{\partial u}{\partial t}(1, t) = t^2 e^t.$$ 

Find $u(0.5, 2)$.

7. (Ex. 1.6.4) Find which of the following operators $L$ are linear:

a) $L[u] = \frac{\partial u}{\partial t} + x^2 \frac{\partial^2 u}{\partial x^2}$

b) $L[u] = \frac{\partial u}{\partial t} + u \frac{\partial^2 u}{\partial x^2} + u$

c) $L[u] = \left(\frac{\partial u}{\partial t}\right)^2 + \frac{\partial^2 u}{\partial x^2}$

d) $L[u] = \frac{\partial^2 u}{\partial t^2} - e^{x^2 t} \frac{\partial^2 u}{\partial x^2} + t^2 u.$