1. (Ex. 1.6.5) Show that the problem
\[
\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0 \quad \text{for} \quad 0 < x < l \quad t > 0
\]
\[
u(x, 0) = f_1(x) \quad \text{for} \quad 0 \leq x \leq 1
\]
\[
\frac{\partial u}{\partial t}(x, 0) = f_2(x) \quad \text{for} \quad 0 \leq x \leq 1
\]
\[
u(0, t) = f_3(t) \quad \text{for} \quad t > 0
\]
\[
\frac{\partial u}{\partial x}(l, t) = f_4(t) \quad \text{for} \quad t > 0.
\]
has at most one solution\(^1\).

2. (Ex. 1.7.2) Find the domain of dependence of the point (0.25, 3) with respect to the problem
\[
\frac{\partial^2 u}{\partial t^2} - (1 + x^2)^2 \frac{\partial^2 u}{\partial x^2} = F(x, t) \quad \text{for} \quad 0 < x < 1 \quad t > 0
\]
\[
u(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1
\]
\[
\frac{\partial u}{\partial t}(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1
\]
\[
u(0, t) = 0 \quad \nu(1, t) = 0 \quad \text{for} \quad t > 0.
\]

3. (Ex. 1.7.5) Show that if \(A(x, t)\) and \(C(x, t)\) are positive continuous differentiable functions \(A\) is nondecreasing and \(C\) is nonincreasing, the initial-value problem
\[
\frac{\partial}{\partial t} \left( A \frac{\partial u}{\partial t} \right) - \frac{\partial}{\partial x} \left( C \frac{\partial u}{\partial x} \right) - F(x, t) \quad \text{for} \quad 0 < x < 1 \quad t > 0
\]
\[
u(x, 0) = f(x) \quad \text{for} \quad 0 \leq x \leq 1
\]
\[^1\text{Note that in the book 2-3 equations need to be switched with 4-5 equations.}\]
\[
\frac{\partial u}{\partial t}(x, 0) = g(x) \quad \text{for} \quad 0 \leq x \leq 1
\]

\[u(0, t) = 0 \quad u(1, t) = 0 \quad \text{for} \quad t > 0.
\]

has at most one solution. Show how to construct the domain of dependence of a point \((\bar{x}, \bar{t})\).

4. (Ex.1.7.2) Show that the domain of dependence of the point \((0.25, 3)\) with respect to the problem

\[
\frac{\partial^2 u}{\partial t^2} - c((x) \frac{\partial^2 u}{\partial x^2} = F(x, t) \quad \text{for} \quad 0 < x < 1 \quad t > 0
\]

\[u(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1
\]

\[\frac{\partial u}{\partial t}(x, 0) = 0 \quad \text{for} \quad 0 \leq x \leq 1
\]

\[u(0, t) = 0 \quad \frac{\partial u}{\partial x}(1, t) + u(1, t) = 0 \quad \text{for} \quad t > 0.
\]

is again bounded by the characteristics \(C_1\) and \(C_2\) satisfying formula (7.7) in section 1.7 of the book.