Due Tuesday, November 11, 2008.

(1) Consider the two following two planes $P_1$ and $P_2$ and the two points $O$ and $p$:

\[
O := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]
\[
p = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}
\]
\[
P_1 = \{x + y + z = 1\}
\]
\[
P_2 = \{x - y - z = 0\}.
\]

Find the points on $P_1$ and $P_2$ closest to $O$ and $p$ respectively.

(2) Suppose that $P_1$ and $P_2$ are planes defined implicitly by row vectors $\rho_1$ and $\rho_2$ respectively: that is

\[
\rho_i := [a_i b_i c_i - d_i]
\]

corresponds to the plane $a_i x + b_i y + c_i z = d_i$, using the vector

\[
\vec{v} := \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}, \quad \rho \vec{v} = 0.
\]

(a) Write a computer program for determining whether $P_1$ and $P_2$ intersect (in terms of $\rho_i$) and computing their line of intersection.

(b) Write a computer program for parametrizing $P_i$ explicitly

\[
\vec{p}_i(s, t) := \vec{u}_i + s\vec{v}_i + t\vec{w}_i
\]

for $s, t \in \mathbb{R}$.

(c) Write a computer program for computing the perspective mapping from $P_1$ to $P_2$.  

\[\text{Date: November 5, 2008.} \]
(3) Consider the one-sheeted hyperboloid
\[ \mathcal{H} := \{x^2 + y^2 = z^2 + 1\}. \]

(a) Find all the lines on \( \mathcal{H} \).

(b) Show that the lines fall into two families \( L_1 \) and \( L_2 \) such that any two lines in the same \( L_i \) are skew and any pair of lines, one from \( L_1 \), and one from \( L_2 \), intersect.

(c) If \( \ell \) is a line defined parametrically as
\[ \vec{p} + \mathbb{R}\vec{v} \]
and \( R_\theta \) is the rotation through angle \( \theta \) about the \( z \)-axis, describe the line \( R_\theta(\ell) \).

(d) Generate \( \mathcal{H} \) by revolving a line around the \( z \)-axis \( x = y = 0 \).

(e) Illustrate these ideas with a computer program.