Math 431 Project Ideas

1. Draw the Helix described in Homework 2, Problem 3. First in an affine patch near the origin, then in a neighborhood of the ideal point, and maybe a few other affine patches.

2. Demonstrate the a cone is projectively equivalent to a cylinder by drawing the cone $x^2 + y^2 - z^2 = 0$, in a variety of affine patches.

3. The cone above contains many straight lines. Draw the cone and some of the lines on it. Perhaps show how the cone can be generated by starting with an appropriate line, and rotating that line around the z-axis.

4. Sketch a triangle (or smiley face or something) and its image under the map: $[X, Y, Z] \rightarrow [-X, -Y, Z]$ in a few different affine patches of $\mathbb{R}P^2$. Maybe animate?

5. Demonstrate that $\mathbb{R}P^2$ is nonorientable: Sketch some parallel lines, maybe $p(t) = (t, 0)$ and $q(t) = (t, 1)$ first in a neighborhood of the origin, then in some other affine patches. Maybe add something similar to Dr. Stickler in Indra’s Pearls and try animate to see how orientation reverses?

6. Demonstrate that circles, parabolas, and hyperbolas are all projectively equivalent by sketching the homogeneous equation $X^2 + Y^2 - Z^2 = 0$ in a variety of affine patches in $\mathbb{R}P^2$. 