1. Ex. 18, p.242 Strang.
2. Ex 4, p.263 Strang.
3. Ex. 8, p.276 Strang.
4. Ex. 15 p.308 Strang. (Do it by hand. You may use MATLAB as a check.)
5. (MATLAB) “All Gaul is divided into three parts.” Call them East Gaul, Central Gaul and West Gaul. Each year 5% of the residents of East Gaul move to Central Gaul and 5% move to West Gaul. Of the residents of Central Gaul 15% move to East Gaul and 10% move to West Gaul. And of the residents of West Gaul, 10% move to East Gaul and 5% move to Central Gaul. What percentage of the population resides in each of the three regions after a long period of time?
6. (MATLAB) Solve $x' = Ax$, $x(0) = x_0$ where
   \[ A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 6 \\ -4 \\ 4 \end{bmatrix}. \]
7. Find the general solution of $x' = Bx$ where
   \[ B = \begin{pmatrix} 0 & 4 \\ -1 & -4 \end{pmatrix} \]
8. Solve $x' = Cx$, $x(0) = x_0$ where
   \[ C = \begin{pmatrix} 5 & -5 \\ 1 & 1 \end{pmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}. \]
9. A matrix $A$ is \textbf{nilpotent} if $A^k = 0$ for some positive integer $k$. Show that if $A$ is nilpotent its only eigenvalue is zero. (Note: The converse of this is also true.)
10. (a) Suppose $A$ is a diagonalizable matrix, all of whose eigenvalues have absolute value less than 1. Show that $A^n \to 0$ as $n \to \infty$.
   (b) Let $A$ be as in part (a). Show that $I - A$ is invertible and
   \[ (I - A)^{-1} = I + A + A^2 + A^3 + \cdots. \]
   (c) Let
   \[ A = \begin{pmatrix} .2 & -.2 \\ .3 & .7 \end{pmatrix}. \]
   Use MATLAB to sum the series and verify the result of part (b). For this set $I = \text{eye}(2), S = I, S = I + S \ast A$. Now by using the “up-arrow” repeat the last command until you have convergence. Compare with $(I - A)^{-1}$. 