1. Let

\[ A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 3 & 2 & 1 \end{pmatrix} \]

(a) Find a permutation matrix \( P \), a lower triangular matrix \( L \) and an upper triangular matrix \( U \) such that

\[ PA = LU. \]  

(b) Show how the factorization (1) is used to solve \( Ax = b \).

2. Let \( u = (1, 1, 1)^T \), \( v = (1, 7, 1, 7)^T \), \( W = \text{Span}\{u, v\} \).

   (a) Calculate \( \|v\| \), the projection of \( v \) onto \( u \) and the unit vector in the direction of \( u \).

   (b) Apply the Gram-Schmidt process to \( \{u, v\} \) to obtain an orthonormal basis for \( W \).

   (c) Let \( y = (3, 2, -1, 2)^T \). Find \( z \), the vector in \( W \) which is closest to \( y \).

3. Let \( u_1 = (1, 2, 3)^T \), \( u_2 = (1, 1, -1)^T \), \( W = \text{span}\{u_1, u_2\} \). Find a basis for \( W^\perp \).

4. Let \( u \) be a unit vector in \( \mathbb{R}^n \) and let \( P = I - 2uu^T \). (\( P \) is an \( n \times n \) matrix.) Show

   (a) \( P \) is symmetric.

   (b) \( P \) is orthogonal.

   (c) \( P^2 = I \).

   If \( u = \left(\frac{3}{5}, \frac{4}{5}\right)^T \), what is \( P \) ?

5. We wish to fit the data (0,1), (1,3), (2,7), (3,10), (4,20) to a function of the form

\[ f(x) = a + bx + cx \]

in the sense of least squares. Find an equation for the coefficients \( a \), \( b \) and \( c \). Do not do any computations.

6. In \( C[0,1] \) with the inner product defined by

\[ f \cdot g = \int_0^1 f(x)g(x) \, dx \]

consider the vectors 1 and \( x \).

   (a) Determine the projection \( p \) of 1 onto \( x \) and verify that \( 1 - p \) is orthogonal to \( p \).

   (b) Compute \( \|1-p\|, \|p\|, \|1\| \) and verify that the Pythagorean theorem holds.