1. (a) Give an example of an infinite collection of open sets \( U_k, k \in \mathbb{N} \) in \( \mathbb{R} \) such that \( \bigcap_k U_k \) is not open.
(b) Give an example of an infinite collection of closed sets \( F_k, k \in \mathbb{N} \) in \( \mathbb{R} \) such that \( \bigcup_k F_k \) is not closed.

2. Let \( A \) be a subset of \( \mathbb{R}^n \). The *characteristic function* of the set \( A \) is the function \( f : \mathbb{R}^n \to \mathbb{R} \) defined by

\[
f(u) = \begin{cases} 
1 & u \in A \\
0 & u \notin A
\end{cases}
\]

Prove that this characteristic function is continuous at each interior point of \( A \) and at each exterior point of \( A \) but fails to be continuous at each boundary point of \( A \).

3. Let \( f(x, y) = |x^2y + x^3y^3|^{1/4} \).
(a) Prove that \( f \) is continuous at \( (0,0) \).
(b) Compute the partial derivatives of \( f \) at \( (0,0) \).
(c) Show directly from the definition that \( f \) is not differentiable at \( (0,0) \).

4. Let

\[ f(x, y) = \frac{1}{x} + \frac{1}{y} + xy. \]

(a) Show that \( (1,1) \) is a critical point of \( f \), i.e. the derivative vector of \( f \) vanishes there.
(b) Decide whether \( (1,1) \) is a local maximizer, a local minimizer or a saddle point for \( f \).