1. Consider $f : \mathbb{R}^n \to \mathbb{R}$ such that $f(p_1 + p_2) = f(p_1)f(p_2)$ for all $p_1, p_2 \in \mathbb{R}^n$.
   (a) Show that either $f(0) = 0$ or $f(0) = 1$.
   (b) Give a (non-constant) example of such an $f$ with $f(0) = 1$ when $n = 1$.
   (c) Assume $f(0) = 1$. Show that $f$ is continuous on $\mathbb{R}^n$ if and only if $f$ is continuous at $0$.

2. Consider $f(x, y) = |x - y|$. Determine all directions $p$ for which $\frac{\partial f}{\partial p}(1, 1)$ exists.

3. Consider $f$ defined by

   $f(x_1, \ldots, x_n) = \begin{cases} 
   x_1x_2\cdots x_n/(x_1^2 + x_2^2 + \cdots + x_n^2), & (x_1, \ldots, x_n) \neq (0, \ldots, 0) \\
   0, & (x_1, \ldots, x_n) = (0, \ldots, 0) 
   \end{cases}$

Determine all values of $n$ for which $f$ is differentiable at the origin.

4. (Rolle’s Theorem) Suppose $f : \mathbb{R}^n \to \mathbb{R}$ is a differentiable function such that $f(p) = 0$ if $\|p\| = 1$. Show that there is a $q$ with $\|q\| < 1$ such that $Df(q) = 0$. (For $n = 1$, this is Rolle’s theorem. Examine the proof in this case.)

5. Ex.15, p.379, Fitzpatrick.


8. Ex.11, p.393, Fitzpatrick.