1.  
(a) Implement the bisection method in MATLAB to find the smallest positive root of
\[ e^{-x} = \sin x \]  
(b) Solve (1) using the secant method. (Use either a calculator or MATLAB.)

2. Write a MATLAB function Newton(f, df, x, tol) to implement Newton’s method. You need to supply functions \( f(x) \) and \( df(x)(f'(x)) \). The input \( x \) is the initial guess and \( tol \) is the desired accuracy which should be attained when \( |x_{i+1} - x_i| < tol \). You should limit the number of iterations and report a failure to converge. Use the \texttt{error} function.
   (a) Try your function to solve equation (1). Print out the iterates and the function values.
   (b) Use your function to find the first ten positive solutions of
   \[ x = \tan x. \]
   Note: The careful selection of \( x \) is critical.
   (c) Try the function on the double root \( x = 2 \) of
   \[ x^3 - x^2 - 8x + 12 = 0. \]
   Use \( x = 3 \) and \( tol = 10^{-6} \). What is the rate of convergence?
   (d) Newton can be used to find complex roots also. By starting with a non-real initial guess, find the complex roots of
   \[ x^3 + 2x - 5 = 0. \]

3. Write down two fixed point procedures for finding a zero of the function \( f(x) = 2x^2 + 6e^{-x} - 4 \). Check that they converge.


5. Consider the mapping \( g(x) = cx(1-x) \).
   (a) Show that for \( 0 \leq c \leq 4 \), \( g \) maps \([0,1]\) into itself.
   (b) Show that if \( 1 < c < 3 \), \( g \) has a positive fixed point which is attracting.
   (c) For \( c = 3.2 \) investigate the dynamics of the iteration \( x_{n+1} = g(x_n) \). Certain numbers play a significant role here. Identify them as fixed points of some mapping.

6. Ex. 8, p.107, Atkinson & Han.


8. Ex.18, p.109, Atkinson & Han.
10. Consider the system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix}
4 & -1 & 0 & -1 & 0 \\
-1 & 4 & -1 & 0 & -1 \\
0 & -1 & 4 & -1 & 0 \\
-1 & 0 & -1 & 4 & -1 \\
0 & -1 & 0 & -1 & 4
\end{pmatrix}$$

and $\mathbf{b} = (-2, -1, 6, 7, 14)'$. Solve the system using

(a) The Cholesky factorization of $A$ (MATLAB: CHOL)
(b) Jacobi iteration.
(c) Gauss-Seidel iteration. (The MATLAB command TRIL might be useful.)