This exam has 8 questions.

Instructions: Number the answer sheets from 1 to 8. Fill out all the informations at the top of each sheet (write and sign the Honor Pledge on page 1 only). Answer one question on each sheet in the correct order. (Do not answer one question on more than one sheet. Use the back of the correct sheet if you need more space).

None of the following are allowed: lecture notes, book, electronic devices of any kind (including calculators, cell phones, etc.)

You may keep with you one sheet of handwritten notes.

1. [25 points] Consider the line $\ell_0$ with symmetric equations

$$\frac{x + 2}{2} = \frac{y - 4}{3}, \quad z = -1$$

and the line $\ell_1$ with symmetric equations

$$\frac{x - 1}{3} = \frac{y - 2}{-2} = \frac{z - 2}{3}$$

(a) Show that $\ell_0$ and $\ell_1$ are perpendicular.
(b) Find the point of intersection $P_0$ of the lines $\ell_0$ and $\ell_1$.
(c) Find an equation of the plane containing both $\ell_0$ and $\ell_1$.

2. [25 points] Consider the curve $C$ parametrized by

$$\mathbf{r}(t) := 2t \mathbf{i} + t^2 \mathbf{j} + \ln t \mathbf{k}, \quad 1 \leq t \leq 4.$$  

(a) Explain why $C$ is a smooth curve
(b) Find parametric equations for the line passing through $P_0 := \mathbf{r}(2)$ in the direction of the tangent to $C$ at $P_0$.
(c) Find the length of the curve $C$.

3. [25 points] Consider the function

$$f(x, y) := x^2 y - x^2 - 2y^2 + 3$$

(a) Find all critical points of the function $f$
(b) Determine whether each critical point yields a relative maximum value, a relative minimum value or a saddle point.
4. [20 points] Let

\[ f(x, y, z) := z e^{x^2 - y^2} \]

(a) Find the directional derivative of \( f \) at the point \((1, 1, 1)\) in the direction of

\[ \vec{a} := 2\vec{i} - \vec{j} - 2\vec{k}. \]

(b) Find the unit vector for which the directional derivative is maximal at the point \((1, 1, 1)\).

5. [30 points] Compute the double integral

\[ \iint_R \frac{1}{1 + x^4} \, dA \]

where \( R \) is the triangle with vertices \((0, 0), (1, 0)\) and \((1, 1)\). Hint: Integrate first in the \( y \) variable.

6. [25 points] Compute the triple integral

\[ \iiint_D (x^2 + y^2) \, dV \]

where \( D \) is the region bounded above by the paraboloid \( z = 25 - x^2 - y^2 \) and below by the \( x-y \) plane.

7. [25 points] Use Green’s theorem to compute the line integral

\[ \oint_C \vec{F} \cdot \, d\vec{r} \]

where \( \vec{F} := y^2\vec{i} + x^2\vec{j} \) and \( C \) is the triangle with vertices \((0, 0), (1, 0), (0, 1)\) oriented counterclockwise.

8. [25 points] Use Gauss’s theorem (also known as the divergence theorem) to compute the flux

\[ \iint_S (\vec{F} \cdot \vec{n}) \, dS \]

where \( \vec{F} := x^2\vec{i} + y^2\vec{j} + z^2\vec{k} \) and \( S \) is the boundary of the region \( D \) bounded above by the sphere \( x^2 + y^2 + z^2 = 9 \) and below by the \( x-y \) plane. \( \vec{n} \) is the unit normal vector directed outward from the region \( D \).