Basic Outline

1.1 Mathematical induction, the Completeness Axiom, infimum, supremum, maximum, minimum, upper bound, lower bound.

1.2 Archimedian Property, dense subset: \( \forall (a, b) \exists x \in S \cap (a, b) \).
   Theorem: \( \mathbb{Q} \) and \( \mathbb{R} \setminus \mathbb{Q} \) are dense in \( \mathbb{R} \).

2.1 Sequences, convergence, Comparison Lemma.

2.2 Bounded sequence, closed subset, sequentially dense: \( \forall x_0 \in \mathbb{R} \exists \{x_n\} \) in \( S \) ct \( x_0 \)
   Theorem: Every convergent sequence is bounded.

2.3 Monotone sequence, Monotone Convergence Theorem, Nested Interval Theorem.
   Theorem (MCT): A monotone sequence converges iff it is bounded and if so to sup or inf.

2.4 Sequential compactness: \( \forall \{x_n\} \) in \( S \) \( \exists \{x_{n_k}\} \) which converges.
   Theorem: Every sequence has a monotone subsequence.
   Theorem: Every bounded sequence has a convergent subsequence.
   Theorem (SCT): \([a, b]\) is sequentially compact.

3.1 Continuity of a function.

3.2 Extreme Value Theorem. Proof contains some critical techniques.

3.3 Intermediate Value Theorem.

3.4 Uniform Continuity
   Theorem: A uniformly continuous function \( f : D \to \mathbb{R} \) is continuous.
   Theorem: A continuous function \( f : [a, b] \to \mathbb{R} \) is uniformly continuous.

3.5 The \( \epsilon-\delta \) criteria.
   Theorem: \( \epsilon-\delta \) at \( x_0 \) iff continuous at \( x_0 \).
   Theorem: \( \epsilon-\delta \) on \( D \) iff uniformly continuous on \( D \).

3.6 Monotone functions, one-to-one, inverses.
   Theorem: If \( f : D \to \mathbb{R} \) is monotone and \( f(D) \) is an interval then \( f \) is continuous.
   Theorem: If \( f : I \to \mathbb{R} \) is strictly monotone then \( f^{-1} : f(I) \to \mathbb{R} \) is continuous.

3.7 Limit points, limits of functions.

How to Study

1. Definitions: Practice showing that things either do or do not satisfy definitions. This can be done either straight from the definition or via a theorem. Consider whether definitions are subsets of one another.

2. Theorems: Determine if certain specific examples fit the hypotheses for a theorem and determining the consequences. Determine if the converse of a theorem is true and if not then finding a counterexample. Determine if all hypotheses are required and if not then finding a counterexample.

3. Together: Analyzing how definitions and theorems fit with each other.