1. True or false: Determine if the following are true or false. If false provide a counterexample. [15 pts]
   (a) A subsequence of a monotone sequence is also monotone.
   (b) A subsequence of an unbounded sequence cannot converge.
   (c) A function \( f : (1, 3) \rightarrow \mathbb{R} \) must be bounded.
   (d) A continuous function \( f : [a, b] \rightarrow \mathbb{R} \) has \( f(\mathbb{R}) \) bounded.
   (e) A bounded subset of \( \mathbb{R} \) must have a maximum.

2. Prove that for \( b > 0 \) and \( n \in \mathbb{N} \) we have \( \left( \frac{1}{2} + b \right)^n \geq b^{n-1} \left( b + \frac{1}{2} n \right) \). [10 pts]

3. Using only the Archimedean Principle give a direct proof that \( \left\{ 7 + \frac{1}{n} + \frac{2}{\sqrt{n}} \right\} \) converges to 7. [10 pts]

4. Define \( f : \mathbb{R} \rightarrow \mathbb{R} \) by \( f(x) = x^2 + 2 \). Verify the \( \epsilon - \delta \) criterion for \( f(x) \) at \( x = 3 \). [15 pts]

5. Prove that the function
   \[
   f(x) = \begin{cases} 
   x^2 & \text{if } x < 1 \\
   x - 1 & \text{if } x \geq 1 
   \end{cases}
   \]
   is continuous at \( x = 2 \) but not at \( x = 1 \). [15 pts]

6. Prove that the function \( f : (0, 1) \rightarrow \mathbb{R} \) given by \( f(x) = \frac{1}{x-1} \) is not uniformly continuous. [10 pts]

7. Prove that a nonnegative convergent sequence must converge to a nonnegative value. [10 pts]

8. Suppose \( \{b_n\} \) is a bounded nonnegative sequence and \( 0 \leq r < 1 \). Define
   \[
   s_n = b_1 r + b_2 r^2 + b_3 r^3 + \ldots + b_n r^n
   \]
   Prove that \( \{s_n\} \) converges. Hint: Use the Monotone Convergence Theorem. [15 pts]

The End