1. For each of the following $f$ defined on $[a, b]$ first find a formula for $F(x) = \int_a^x f(t) \, dt$ for $x \in [a, b]$ which does not involve integrals. Then sketch the graph of $F(x)$.

(a) $f : [0, 10] \to \mathbb{R}$ defined by
   \[
   \begin{cases}
   x + 1 & \text{for } x \in [0, 5] \\
   6 & \text{for } x \in (5, 10]
   \end{cases}
   \]

(b) $f : [-1, 1] \to \mathbb{R}$ defined by
   \[
   \begin{cases}
   2 & \text{for } x \in [-1, 0] \\
   x + 1 & \text{for } x \in (0, 1]
   \end{cases}
   \]

2. One of the previous problem’s $F(x)$ is not an antiderivative of the $f$. Identify which is not and explain why not.

3. Suppose that $f : \mathbb{R} \to \mathbb{R}$ has continuous second derivative. Prove that
   \[
   f(x) = f(0) + f'(0)x + \int_0^x (x - t)f''(t) \, dt \quad \text{for all } x
   \]
   Note: There’s a hint somewhere in the book.

4. Suppose that $f : \mathbb{R} \to \mathbb{R}$ has a second derivative and that
   \[
   f''(x) - f'(x) + xf(x) = x^2 \\
   f(1) = 3 \\
   f'(1) = -2
   \]

   (a) Find and simplify $P_3(x)$ at $x_0 = 1$.

   (b) Use your answer to (a) to approximate $f(1.1)$. Simplify.

   (c) Use your answer to (a) to approximate $\int_{0.9}^{1.1} f$. Simplify.

5. Prove that
   \[
   1 + \frac{x}{3} - \frac{x^2}{9} < (1 + x)^{1/3} < 1 + \frac{x}{3} \quad \text{for } x > 0
   \]

6. Suppose that each of $f, g : \mathbb{R} \to \mathbb{R}$ has $n + 1$ continuous derivatives. Prove that $f$ and $g$ have contact of order $n$ at $x = 0$ if and only if
   \[
   \lim_{x \to 0} \frac{f(x) - g(x)}{x^n} = 0
   \]