1. Prove that the function $f : (1, \infty) \to \mathbb{R}$ given by $f(x) = \frac{x}{1-x}$ is strictly monotone.

2. Is the converse of Theorem 3.23 true or false? Justify.

3. Let $D = [0, 1] \cup [2, 3]$ and define $f : D \to \mathbb{R}$ by

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ x^2 - 3 & \text{if } 2 \leq x \leq 3 \end{cases}$$

(a) Prove that $f$ is continuous using Theorem 3.23.

(b) Prove that $f$ is continuous using the definition of continuity.

4. Define $f : [1, 4] \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2x + 1 & \text{if } 1 \leq x \leq 3 \\ x^2 & \text{if } 3 < x \leq 4 \end{cases}$$

(a) Find $f([1, 4])$.

(b) Find a formula for $f^{-1} : f([1, 4]) \to [1, 4]$.

(c) Prove that $f^{-1}$ is continuous.

5. Find $\lim_{x \to 2} \frac{x^2 - 2}{x^4 - 16}$.

6. Prove that $\mathbb{N}$ has no limit points.