Math 131 Exam 1 Sample 2

Directions: Do not simplify unless indicated. Non-graphing calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem. All real-world problems should include units.

Please put problem 1 on answer sheet 1

1. (a) Use Simpson’s rule to approximate \( \int_{1}^{4} \frac{1}{x+1} \, dx \) with \( n = 6 \) subintervals. [10 pts]

(b) Suppose we are looking at waitresses and cooks (W and C) working at three different restaurants (R1, R2 and R3) during three different weeks (W1, W2 and W3). Consider the following two tables. The first contains the total amount (in hours/week) that all waitresses and cooks worked in each of the three weeks. These amounts are the same for each restaurant. The second contains the salaries (in $/hour) of waitresses and cooks at the three restaurants.

<table>
<thead>
<tr>
<th>hours/week</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>200</td>
<td>250</td>
<td>220</td>
</tr>
<tr>
<td>C</td>
<td>250</td>
<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$/hour</th>
<th>W</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>R2</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>R3</td>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

i. What does the 15 represent? [2 pts]

ii. What does the 220 represent? [2 pts]

iii. Put these tables into matrices and multiply them in such a way that the result is meaningful. Describe what the resulting matrix represents. [6 pts]

Please put problem 2 on answer sheet 2

2. (a) Evaluate \( \int_{1}^{4} e^{x} \, dx \). [10 pts]

(b) Find \( P \) so that the average value of \( f(x) = Pe^{0.05x} \) on \([0, 10]\) equals 5000. [10 pts]

Please put problem 3 on answer sheet 3

3. (a) Evaluate \( \int_{2}^{\infty} \frac{2x}{(x+1)^{1/2}} \, dx \). [15 pts]

(b) If \( A \) and \( B \) are matrices and both \( AB \) and \( BA \) make sense, must they both be square? Explain. [5 pts]

Please put problem 4 on answer sheet 4

4. Consider the system of equations

\[
\begin{align*}
x + y &= 1 \\
3x - y &= 7
\end{align*}
\]

(a) Use Gauss-Jordan to solve the system. [10 pts]

(b) Use the inverse of a matrix to solve the system. [10 pts]

Please put problem 5 on answer sheet 5

5. (a) Find the inverse of the matrix

\[
\begin{bmatrix}
1 & -2 & 1 \\
0 & 1 & 2 \\
0 & 0 & 2
\end{bmatrix}
\]

[5 pts]

(b) Suppose the Leslie matrix for a population of juveniles and adults is shown below. Find a population of 4000 for which the ratio of juveniles to adults stays constant over time. By how much is the total population shrinking?

\[
\begin{bmatrix}
0.3 & 0.5 \\
0.5 & 0.3
\end{bmatrix}
\]

[15 pts]

The End