1. (a) [10pts] True or False. **Justify** your answers carefully.
   i. If a matrix $A$ is orthogonally diagonalizable, then $A$ is symmetric.
   ii. The solution of a least-squares problem is always unique.
   iii. If an $n \times n$ matrix has $n$ distinct eigenvalues, then it is diagonalizable.
   iv. If two matrices are row equivalent, then they have the same eigenvalues.

   (b) [15pts] Let $A$ be a $2 \times 2$ matrix given by $A = PDP^{-1}$ where
   \[
P = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} \text{ and } D = \begin{bmatrix} .5 & 0 \\ 0 & 2 \end{bmatrix}.
   \]
   Let $\vec{x}_k$ be solution of the dynamical system $\vec{x}_{k+1} = A\vec{x}_k$, with $\vec{x}_0$ such that
   \[
P^{-1}\vec{x}_0 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.
   \]
   i. Compute $\vec{x}_1 = A\vec{x}_0$.
   ii. Find a formula for $\vec{x}_k$ for all $k$, involving the vectors $\vec{u}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\vec{u}_2 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$ (which are the columns of $P$).

2. [25pts] Consider the matrix $A = \begin{bmatrix} 2 & -3 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.
   (a) Show that the eigenvalues of $A$ are $\lambda = 2$ and $\lambda = -1$.
   (b) Diagonalize $A$ (give the matrices $P$ and $D$. You do not need to compute $P^{-1}$).

3. (a) [15pts] Let $A = \begin{bmatrix} 1 & 2 \\ -3 & 3 \end{bmatrix}$. Find the (complex) eigenvalues and corresponding (complex) eigenvectors of $A$.
   (b) [10pts] Consider the quadratic form $Q(\vec{x}) = 3x_1^2 + x_2^2 + 4x_1x_2$. Find the symmetric matrix $A$ such that $Q(\vec{x}) = \vec{x}^T A \vec{x}$ and classify the quadratic form $Q$ as positive definite, negative definite or indefinite.
4. Let $W = \text{Span} \{ \vec{u}_1, \vec{u}_2 \}$ where $\vec{u}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ and $\vec{u}_2 = \begin{bmatrix} 4 \\ -6 \\ 1 \end{bmatrix}$.

(a) [10pts] Find an orthogonal basis of $W$.

(b) [10pts] Find the projection of the vector $\vec{b} = \begin{bmatrix} 10 \\ 5 \\ 9 \end{bmatrix}$ onto $W$.

(c) [5pts] Find the distance from $\vec{b}$ to $W$. 
