ERRATA AND SUGGESTION SHEETS
Advanced Calculus, Second Edition

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• Page 11, line 38: “$b^2 < r$” should be “$b^2 < c$”
• Page 13, line 1: for “... number $a$ and $b$,” write “... numbers $a$ and $b$,”
• Page 16, 1c: “$\mathbb{Q} \setminus \mathbb{N}$” should read “$\mathbb{Q} \setminus \mathbb{Z}$”
• Page 18, line 14: “$1 - T$” should read “$1 - r$.”
• Page 19: The Geometric sum formula. Left hand side is not define when $r = 0$.
• Pages 30, 32: To slightly improve clarity, the Linearity Property should come before Theorem 2.13.
• Page 32, line 16: “$p(x) = \sum_{i=0}^{k} c_i x^i$” should appear “$p(x) = c_0 + \sum_{i=1}^{k} c_i x^i$”
• Page 36, line 16: “$s = (a + b)/2$” should be “$s = (a + b)/2$.”
• Page 38: Theorem 2.25 (i) and (ii) should have $n$ in NATURAL numbers $\mathbb{N}$.
• Page 40, line 2: “$S_4 + \frac{1}{2} = 1 + \frac{3}{2}$” read “$S_4 + \frac{1}{2} \geq 1 + \frac{3}{2}$.”
• Page 54, line 9: “sequence $\{-1/n\}$” should read “sequence $\{-1/n\}$”
• Page 57, line 14: “... $f + g : \mathbb{R} \rightarrow \mathbb{R}$ and ...” should read “... $f + g : \mathbb{R} \rightarrow \mathbb{R}$ and ...”
• Page 62, line 7: “the value 0.” should read “zero or negative values.”
• Page 67, line 21: “$1/n$” should read “$-1/n$.”
• Page 67, line 22: “$2 + 1/n^2$” should read “$-2 - 1/n^2$.”
• Page 67, line 24: “$(0, 1)$” should read “$(0, 2)$.”
• Page 73: Theorem 3.22 in the first sentence after “ii”, it reads “... criterion at the domain $D$; ...” and it should read “... criterion on the domain $D$; ...”
• Page 78, line 22: “monotonically increasing” should read “monotone”
• Page 81, line 23: “$\mathbb{D}$” should appear “$\mathbb{D}$.”
• Page 90, line 14: “$\lim_{x \to 0, x > 0} \frac{f(x) - f(0)}{x} = -1$” should read “$\lim_{x \to 0, x < 0} \frac{f(x) - f(0)}{x} = -1$.”
Page 90, line 1 and Page 91, line 1: 
“\(a + x_0^{n-2} + x_0^{n-1}\) should read “\(a + xx_0^{n-2} + x_0^{n-1}\)”.

Page 94, #3: The function value \(f(0)\) is defined twice.

Page 99, (4.8): “\(x - x\)” should read “\(x - x_0\)” in two denominators.

Page 107, line 8: “\(x_0 < x_0 + \delta\)” should appear “\(x_0 < x < x_0 + \delta\)”.

Page 107, line 15: for “In section 9.5,” write “In Section 9.6,”

Page 112, line 3: “\(g(n (x_0) = n!\)” should appear “\(g(n (x) = n!\)”.

Page 112, line 14: for “\(f(n (x_0)) = g(n (x_0))\)” write “\(f(n (x_0)) = g(n (x_0))\)”.

Page 120, line 15: for “inverse function \(\mathbb{R}\).” write “inverse function on \(\mathbb{R}\).”

Page 128, line 11: “\(C(2z) \leq 0\)” should appear “\(C(2z) < 0\)”.

Page 142, line 1: for “...1988), a clear ...” write “...1988), is a clear ...”

Page 143: Under 6.14 you should refer to Darboux sums, not Riemann.

Page 144, line 10: the second “(6.19)” should be “(6.20).”

Page 145, line 3: for “[\(a, b\): \(\mathbb{R} \rightarrow \mathbb{R}\)” write “\(f:[a, b] \rightarrow \mathbb{R}\)”

Page 149, #4b.: for “\((b-a)/2\)” write “\((b^2 - a^2)/2\)”

Page 150, line 10: for “The \(f\) is” write “Then \(f\) is.”

Page 152, line 5: for “\(L(f, P_n)\)” write “\(U(g, P_n)\)”.

Page 152, line 12: for “\(\ldots \leq L(f, P) + U(g, P)\)” write “\(\ldots \leq U(f, P) + U(g, P)\)”.

Page 153, line 1: for “\(\ldots \leq U(f + g, P_n) \leq L(f, P_n) + U(g, P_n)\)” write “\(\ldots \leq U(f + g, P_n) \leq L(f, P_n) + U(g, P_n)\)”.

Page 153, line 21: \(U(\alpha f, P_n)\) Formula is in conflict with formulas 6.31. To avoid that, add the following statement: “The above formula is consistent with formula 6.31 because \(U(\alpha f, P) = L(\alpha f, P)\) for all \(P\) if \(\alpha = 0\)”.

Page 156, lines 10–12: for “\([x_{i-1} - x_i]\)” write “\([x_i - x_{i-1}]\)”

Page 160, line 2: for “Section 7.4.” write “Section 7.3.”

Page 162, line 8: for “\(L(f, P)\)” write “\(L(f', P)\)”.

Page 162, line 8: for “\(R(f, P)\)” write “\(U(f', P)\)”.

Page 164, #3: for “\(\int_a^b f = 4\)” write “\(\int_2^6 f = 4\)”

Page 169, line 7: “from bottom(7.2)” should be “(7.1)”

Page 169, line 8: “from bottom(7.3)” should be “(7.2)”
• Page 180: Possible typo: I would delete $H(d) = 0$. Not needed in argument, and not proved. It is really necessary to change 4.19 slightly.

• Page 187, line 11: for “index $i \geq 1$” write “index $i$ such that $1 \leq i \leq n$”

• Page 189, #8: for “Supose” write “Suppose”

• Page 191: In the caption of Figure 7.2: Reads “… trapezoid …” rather than “… trapezoid …”

• Page 201, line 4: for “$x = 0$” write “$x_0 = 0$”

• Page 201, line 8: for “$x = 0$” write “$x_0 = 0$”

• Page 201, line 12: for “$x = 0$” write “$x_0 = 0$”

• Page 201, line 7: for “$x = 1$” write “= 1”

• Page 202, line 10: for “strictly increasing …” write “strictly decreasing …”

• Page 202, line 1: for “at $x = 0$” write “at $x_0 = 0$.”

• Page 203, line 10: for “$(x - x_0)^n$” write “$(x - x_0)^{n+1}$.”

• Page 205, line 3: from bottom “$n > 4$” should appear “$n \geq 4$.”

• Page 206, line 1: for “$\ln(n+1) = \ln 1$” write “$\ln(n+1) - \ln 1.$”

• Page 217, line 6: for “number $n$” write “number $k$”

• Page 221, line 10: for “about $x = 0$” write “about $x_0 = 0.$”

• Page 225, line 6: for “$1 \leq k \leq n$” write “$0 \leq k \leq n.$”

• Page 233, line 2: for “for index” write “for every index”

• Page 235, line 9: for “$(0, c)$” write “$(0, b)$”

• Page 240, line 2: for “$\lim_{n \to \infty} \left( \frac{a_k}{b_k} \right)$” write “$\lim_{k \to \infty} \left( \frac{a_k}{b_k} \right)$”

• Page 241, line 12: for “… value is 1.” write “… value is 1,”

• Page 241, Fig. 9.2: for “… $\lim_{n \to \infty} 1^n = 0.$” write “… $\lim_{n \to \infty} 1^n = 1.$”

• Page 242, line 1: for “… natural number $k$” write “… integer $k.$”

• Page 243, line 6: from bottom “$2/N < x.$”

• Page 243, line 8: for “… number $n, …$” write “number $n \geq 2, …$”

• Page 243, line 9: for “$f_n(0) = f(2/n) = …$” write “$f_n(0) = f_n(2/n) = …$”

• Page 243, line 10: for “and $[2/n, 0]$” write “and $[2/n, 1].$”

• Page 243, Fig. 9.4: for “$(\frac{1}{n}, 1)$” write “$(\frac{1}{n}, n).$”
• Page 251, line 4: for “4[b − a]” write “[4(b − a)]”

• Page 251, line 8: for “6[b − a]” write “[6(b − a)]”

• Page 257, line 4: for “Cauchy on A” write “Cauchy on A”

• Page 265, Fig. 9.6: left figure: for “(l, 2l)” write “(l, l).”

• Page 265, Fig. 9.6: Two comments: (1) It would be nice to use the same script l as in the surrounding text. (2) It would be nice if the graphs had the same scales for both x- and y-axes.

• Page 266, line 16: for “\[\sum_{k=1}^{\infty} h_k(x)\]” write “\[\sum_{k=0}^{\infty} h_k(x)\]”

• Page 279, line 6: for “dist(\(u, u'\)) and” write “dist(\(u, u'\)) = 0 and”

• Page 286, line 1: from bottom “\(\bigcap_{i=1}^{k} c_i\)” should appear “\(\bigcup_{i=1}^{k} c_i\)”

• Page 302, line 7: for “\(A: \mathbb{R} \to \mathbb{R}\)” write “\(f: A \to \mathbb{R}\).”

• Page 324, line 8: for “\(f: \mathbb{R} \to \mathbb{R}\)” write “\(f: I \to \mathbb{R}\).”

• Page 355, line 4: Is “\(e_i\)” defined in the text (other than p. 281, H.W. #2)?

• Page 373, line 11: for “\(\left(\frac{1}{k}\right)\)” write “\(\left(\frac{1}{k}\right)\).”

• Page 375, line 8: for “\(h\)” write “\(h\)”

• Page 391, line 8: for “\(\nabla f(x) = 0\)” write “\(\nabla f(x) = 0\)”

• Page 452, line 3: “Since the point \((x_0, y_0)\) belongs to \(V\)” should be replaced by “Since the point \((x_0, 0)\) belongs to \(V\).”

• Page 474, line 10: the word “integrable” comes before it is defined (p. 475).

• Page 479, lines 14–15: for “in any one of the \(P_k(J)\)’s” write, perhaps, “in all of the \(P_k(J)\)’s”

• Page 479, line 16: for “\(\sum_{J \in p} U(\cdots) - L(\cdots)\)” write “\(\sum_{J \in p} [U(\cdots) - L(\cdots)]\)”

• Page 479, lines 14–21: It does not seem that \(P_k\) can be chosen as indicated. One suggestion is to: Let \(P_k^*\) be the partition of \(I\) induced by the \(P_k(J)\)’s (By this we mean that for all the \(J\)’s in a common “row” of \(P\), we form the union of all the partition points of a common edge of the corresponding \(P_k(J)\)’s. This union then forms one part of the partition \(P_k^*\) for that corresponding edge.) It should be clear that for each \(J\), \(P_k^*(J)\) is a refinement of \(P_k(J)\) so that

\[
U(f, P_k^*(J)) - L(f, P_k^*(J)) \leq U(f, P_k(J)) - L(f, P_k(J))
\]

for all \(J\) and hence
\[ U(f, P^*_k) - L(f, P^*_k) = \sum_J [U(f, P^*_k(J)) - L(f, P^*_k(J))] \]
\[ \leq \sum_J [U(f, P_k(J)) - L(f, P_k(J))] \]
\[ < \frac{m}{mk} \]
\[ = \frac{1}{k} \]

Thus,
\[ \lim_{k \to \infty} [U(f, P^*_k) - L(f, P^*_k)] = 0, \]

And therefore, by the Archimedes-Riemann Theorem, the function \( f \) is integrable on \( I \).

- **Page 479, line 19**: for “\(-L(f, P_k) =\)” write “\(-L(f, P_k)\) =”
- **Page 488, line 5**: for “vol \( J \)” write “vol \( J \)’” (twice).
- **Page 488, line 11**: for “For positive number \( a \) and \( b \), show that the ellipse” write “Show that the set”
- **Page 488, line 7**: for “that the ellipsoid” write “that the set”
- **Page 489, line 6.7**: for “in the interior of \( J \)” write “in the interior of \( I \)”
- **Page 491, line 2**: for “= \( \int_J \hat{J} \)” write “= \( \int_{I_1} \hat{J} \)”
- **Page 493, line 15**: for “\( \{(x, g(x))\} \)” write “\( \{(x, g(x))\} \)”
- **Page 499, line 10**: for “(19.3)” write “(19.1)”
- **Page 500, line 2**: for “of \( m_i \) and \( M_i \)” write “of \( M_i \)”
- **Page 519, Problem 10**: replace “\( R^2 \)” by “\( R^n \)”