(1) Suppose that you are using the explicit (forward) Euler method to solve a differential equation. If you decrease the step size $h$ by a factor of 10, by what factors will the local and global errors decrease?

Solution: Forward Euler is a first order method, so its global error is order 1 and local error is order 2. Therefore the global error will be reduced by a factor of 10 and the local error will be decreased by a factor of $(10)^2 = 100$.

(2) Suppose that a 1,000,000 L (liter) swimming pool is initially at a chlorine concentration of 1 mg/L (mg = milligrams). Due to an unfortunate ‘accident’, there is an unusually high concentration of bacteria in the pool. To remedy this situation, a hose is put in the pool which pumps chlorinated water at a concentration of 50 mg/L into the pool at the rate of 1 L/s. The mixed water is drained at a rate of 0.5 L/s. Set up an initial value problem to describe evolution of the chlorine concentration over time.

Solution: The volume $V(t)$ of the pool will increase at constant rate $V(t) = 1,000,000 + 0.5t$. The mass of chlorine in the pool, $S(t)$ is increasing at a rate of $(50) \times 1 = 50$ mg/s and decreasing at a rate of $(S(t)/V(t)) \times 0.5$ mg/s. Therefore the evolution of $S(t)$ is governed by

$$\frac{dS}{dt} = -\frac{0.5S(t)}{1,000,000 + 0.5t} + 50, \quad S(0) = 1,000,000.$$

The concentration is then $C(t) = S(t)/V(t)$. 