Classify the type of behavior of the following linear planar systems (give one of the names discussed in class), and sketch the behavior in the plane. Be sure to label your axes and any relevant vectors.

(a) \[
\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}
\]

**Solution:** The characteristic polynomial is
\[p(z) = z^2 + 4z + 5 = (z + 2)^2 + 1.\]

Therefore we have complex eigenvalues \( \lambda = -2 \pm i \), since \( \mu = -2 < 0 \) and \( a_{21} = -1 < 0 \) this will be a clockwise spiral sink. The phase diagram is shown below:

(b) \[
\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}
\]

**Solution:** Since the matrix \( A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \) is triangular, you should be able to read the eigenvalues off the diagonal \( \lambda = 2, 3 \). However you can see this from the characteristic polynomial as well,
\[p(z) = z^2 - 5z + 6 = (z - 2)(z - 3).\]
Therefore the eigenvalues are $\lambda = 2, 3$, to find the corresponding eigenvectors, we look at the matrices

$$A - 2I = \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}, \quad A - 3I = \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}.$$ 

Therefore the eigenpairs are

$$\begin{cases} 2, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 3, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \end{cases}.$$ 

Since both eigenvalues are positive and distinct the phase portrait will be that of a nodal source. The representative curves will come out tangent to $(1, 0)$ near the origin since $\lambda = 3$ is the smaller eigenvalue. The phase diagram is shown below: