(1) Let $\gamma(t)$ be a regular curve in $\mathbb{R}^3$ which does not pass through the origin. If $\gamma(t_0)$ is a point on the curve nearest the origin, show that $\gamma(t_0)$ and $\dot{\gamma}(t_0)$ are orthogonal.

(2) Let $\gamma: \mathbb{R} \to \mathbb{R}^3$ be defined by

$$\gamma(t) = (\cosh(t), \sin(t), \cos(t))$$

Find a unit speed reparametrization of $\gamma$.

(3) A circular disk of radius 1 in the $xy$-plane rolls without slipping along the $x$-axis. The curve swept out by a fixed point on the disk’s circumference is called a cycloid (see the picture below).

(a) Obtain a parametrization of the cycloid and determine its singular points.

(b) Compute the arc length of the portion of the cycloid corresponding to one complete rotation of the disk.