MATH 436 – HOMEWORK #3 – DUE OCT 11

(1) Using the inverse function theorem, find all points \((x, y, z) \in \mathbb{R}^3\) such that the function 
\[ f(x, y, z) = (x^2 + y^2, z^2 + z, zy) \]
has a differentiable inverse in a neighborhood of \((x, y, z)\).

(2) Define a function 
\[ f(x, y, z) = x^2 + y^2 + z^2 + xy + xz + yz - x - y - z \]
Show that the equation \(f(x, y, z) = 0\) defines a smooth surface in \(\mathbb{R}^3\).

(3) Show that the paraboloid \(z = x^2 + y^2\) is diffeomorphic to a plane.

(4) Prove that the definition of a differentiable map between surfaces does not depend on the parametrization.

Date: Sep 11, 2012.