Let $S \subset \mathbb{R}^3$ be a regular surface, and $p \in S$.

(1) Show that we may find coordinates $\sigma(u, v)$, $\sigma(0, 0) = p$ such that the first fundamental form:
\[ I = E(u, v)du^2 + 2F(u, v)dudv + G(u, v)dv^2 \]
satisfies $E(0, 0) = G(0, 0) = 1$ and $F(0, 0) = 0$.

(2) Suppose $S$ is given by the coordinate
\[ \sigma(u, v) = \left( u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2 \right) \]
Compute the first and second fundamental forms.

(3) Let $\varphi : S \to \mathbb{R}$ be a smooth function and $\sigma(u, v)$ a coordinate. Let $\Phi = \varphi \circ \sigma$ be the composition, and $\Phi_u$, $\Phi_v$, the partial derivatives. The gradient of $\varphi$ at $p$ is the unique vector $\nabla f(p) \in T_pS$ satisfying
\[ \langle \nabla \varphi(p), X \rangle = D\varphi(p)(X) \]
for all $X \in T_pS$.

(a) Show that in local coordinates:
\[ \nabla \varphi(p) = \frac{\Phi_u G - \Phi_v F}{EG - F^2} \sigma_u + \frac{\Phi_v E - \Phi_u F}{EG - F^2} \sigma_v \]

(b) Suppose $\nabla \varphi(p) \neq 0$. Show that $D\varphi(p)(X)$ is maximized among all unit vectors $X \in T_pS$ if and only if
\[ X = \frac{\nabla \varphi(p)}{||\nabla \varphi(p)||} \]