(a) Let $X$ be a commutative ring with 1, and let $f$ be any element of $X$. Show that the multiplicative group of units of $X$ is a subgroup of $X$. If $f$ is invertible, prove that $f^{-1}$ is also invertible.

(b) Let $R$ be a commutative ring with 1. An element $r \in R$ is said to be nilpotent if there exists some positive integer $n$ such that $r^n = 0$. Show that the set of nilpotent elements of $R$ is a two-sided ideal of $R$. Prove that if $n$ is invertible in $R$, then $R$ is a domain.

(c) Let $R$ be a commutative ring with 1. An element $r \in R$ is said to be nilpotent if there exists some positive integer $n$ such that $r^n = 0$. Show that the set of nilpotent elements of $R$ is a two-sided ideal of $R$. Prove that if $n$ is invertible in $R$, then $R$ is a domain.

(d) Let $R$ be a commutative ring with 1. An element $r \in R$ is said to be nilpotent if there exists some positive integer $n$ such that $r^n = 0$. Show that the set of nilpotent elements of $R$ is a two-sided ideal of $R$. Prove that if $n$ is invertible in $R$, then $R$ is a domain.

(e) Let $R$ be a commutative ring with 1. An element $r \in R$ is said to be nilpotent if there exists some positive integer $n$ such that $r^n = 0$. Show that the set of nilpotent elements of $R$ is a two-sided ideal of $R$. Prove that if $n$ is invertible in $R$, then $R$ is a domain.

(f) Let $R$ be a commutative ring with 1. An element $r \in R$ is said to be nilpotent if there exists some positive integer $n$ such that $r^n = 0$. Show that the set of nilpotent elements of $R$ is a two-sided ideal of $R$. Prove that if $n$ is invertible in $R$, then $R$ is a domain.
6. Let $G$ be a finite group and let $p: \mathbb{C} G \to \mathbb{C}$ be a group representation of $G$. Let $N$ be the set of polynomials $f \in \mathbb{C}[x]$ such that $f(G) = 0$. Prove that all elements of $\mathbb{C}[x]/(f)$ are in the ideal of $\mathbb{C}$ generated by $f$. Show that if $f$ is an irreducible polynomial in $\mathbb{C}[x]$, then $\mathbb{C}[x]/(f)$ is a field.

Let $f(x) \in \mathbb{C}[x]$ be an irreducible polynomial. Let $\mathbb{C}[x]/(f)$ be the vector space of all polynomials in $\mathbb{C}[x]$ modulo $(f)$. We claim that $\mathbb{C}[x]/(f)$ is a field.

(a) Show that $\mathbb{C}[x]/(f)$ is a vector space over $\mathbb{C}$.
(b) Show that $\mathbb{C}[x]/(f)$ is an integral domain.
(c) Show that $\mathbb{C}[x]/(f)$ is a field.

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(a) Show that $\mathbb{C}[x]/(f)$ is a vector space over $\mathbb{C}$.
(b) Show that $\mathbb{C}[x]/(f)$ is an integral domain.
(c) Show that $\mathbb{C}[x]/(f)$ is a field.
Since $P$ is a point of accumulation of $A$, there exists a sequence $(a_n)$ in $A$ such that $a_n \to P$.

Now suppose $P \in \text{cl}(E \cup F)$. Since $P$ is a point of $F$, there exists a sequence $(b_n)$ in $F$ such that $b_n \to P$.

Thus $P$ is a point of accumulation of $F$. By the previous argument, there exists a sequence $(c_n)$ in $F$ such that $c_n \to P$.

Since $P$ is a point of accumulation of both $A$ and $F$, it is a point of accumulation of $A \cup F$. Therefore, $P$ is a point of accumulation of $E$. Hence, $E$ is dense in $F$.
If \( f \) is a polynomial with real coefficients, then any non-real complex root must come in conjugate pairs.

Let \( \alpha = a + bi \) be a complex root of \( f(x) = 0 \). Then \( \bar{\alpha} = a - bi \) is also a root of \( f(x) = 0 \).

So, \( (x - \alpha)(x - \bar{\alpha}) = (x - a - bi)(x - a + bi) \) is a factor of \( f(x) \).}

In fact, \( f(x) = (x - \alpha)(x - \bar{\alpha})g(x) \), where \( g(x) \) is also a polynomial with real coefficients.

Therefore, if \( f(x) \) has a complex root, then it must have a real root as well.

Hence, \( f(x) \) must have \( n \) real roots, where \( n \) is the degree of \( f(x) \).
\[ \text{(Note: This statement is not true for } n = 2) \]
\[ p \quad \Rightarrow \quad q \quad \Rightarrow \quad \neg p \quad \lor \quad q \]

\[ W = \left[ \begin{array}{c} \frac{x^2}{c} \\ 0 \\ 1 \end{array} \right] \]

In the case where \( W \) is a positive definite matrix, we can rewrite the condition as follows:

\[ P \rightarrow \neg P \]

Suppose \( P \) is not positive. Then the condition is not satisfied.

The only assumption with \( x \) in the equation is that \( x \) is also in the equation. But as we know:

\[ p = x \quad \text{and} \quad \neg p = x \quad \text{for} \quad \neg p \]

\[ \neg p = \neg p \quad \text{and} \quad p = \neg p \]

So if \( W \) has a non-zero eigenvalue \( \lambda \) in the equation.