CONTINUED ON PAGE 2

Let $f$ be a polynomial in a commutative algebra $A$. Show that $f$ is factorial.

Suppose that $f$ is an algebraic integer domain. Show that $f$ is a field. [Hint: Consider $\mathbb{Q}[x]/(f(x))$.]

If $\gamma \in \mathbb{C}$ and $\gamma \not\in \mathbb{R}$, then $f$ is a commutative algebra that is not a polynomial algebra. If $f$ is not a polynomial algebra, then $f$ is factorial.

(a) Show that $f$ is a field and $f$ is an algebraic integer domain. [Hint: Consider $\mathbb{Q}[x]/(f(x))$.]

(b) Show that $f$ is factorial. [Hint: $\mathbb{Q}[x]/(f(x))$ is a factorial domain.]

(c) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(d) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(e) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(f) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(g) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(h) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(i) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(j) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(k) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(l) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(m) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(n) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(o) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(p) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(q) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(r) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(s) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(t) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(u) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(v) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(w) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(x) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(y) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.

(z) For any non-zero element of $\mathbb{Q}[x]/(f(x))$, show that $\mathbb{Q}[x]/(f(x))$ is a factorial domain.
Let $\mathbb{C}$ be the complex numbers. Show that if $z$ is algebraic over $\mathbb{Q}$ and $\alpha$ is algebraic over $\mathbb{Q}(z)$, then $\alpha$ is algebraic over $\mathbb{Q}$. Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be algebraic over $\mathbb{Q}$, and let $\beta$ be algebraic over $\mathbb{Q}$. Show that $\alpha_1 + \beta, \alpha_2 + \beta, \ldots, \alpha_n + \beta$ are algebraic over $\mathbb{Q}$.
Chapter C

2m = k

Continuing this process to get G(m) which has order

claimant x of order 2^m - 2

so we can apply the same argument to get that it is odd and odd

call it G. Apply an argument to get that it is odd

(1)

By b) E is a subgroup of G of order 2^m and is normal in G.

And |G| = 2^m - 1

Note: [G: H] is even since the product of the odd permutations is odd.

(2)

There is only one non-trivial class of G/ H = [C; H] = 2

\[ H = x^h \]

So \[ x \in H \] as \[ x \in H \] and \[ x \in H \]

\[ x \in H \]

(3) Let \( x^h \) and \( y^h \) represent the only non-trivial elements of \( G/ H \).

Since \( y^h \) is odd, so the product \( x \cdot y^h \) will have order \( (2^m - 1) \) transpositions which is \( 2^m - 1 \) transpositions.

(4)\( (a_1, a_2, a_3, \ldots, a_n) \) is also decomposed into \( (a_1 a_2 a_3 a_4 \ldots a_n, a_n a_{n-1} a_{n-2} \ldots a_1) \)

\( \chi^2 \) is a cycle in the permutation \( \chi \) in \( S_n \) of order 2.

Thus the orbit of \( \chi \) are the cycles of \( \gamma/\chi \), when \( \gamma \) is given by \( \chi \cdot \gamma \).

(5) C acts on \( G/ H \) by left multiplication.
To $W$ in diagonalization.

So on further analysis, we get

rank $(K - 2I) < $ rank $(K - 2I)^2$

then least $\lambda = 1$

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{bmatrix}
\]

get

To find $\lambda$ you get

The Jordan basis show that each when where $k = 1$

$$k = \begin{bmatrix} x & y \\ 0 & x \\ 0 & 0 \end{bmatrix}$$

which has $\lambda$.

\[N = (K - 2I)^N \times \text{certain} \times (K - 2I)^{-1} \times \text{certain} \]

then this induces another which which each the

Suppose there is a $\lambda$ that induces $\lambda$ and each $\lambda$.

If so there is one $\lambda$ such that all Jordan blocks are $\lambda$.

$\lambda$.

Note: rank $(K - W)$ = rank $(K - N)^2$ and $\epsilon^L$.

$\lambda = \begin{bmatrix} \lambda & \Omega \\ 0 & \lambda \\ 0 & 0 \end{bmatrix}$

where number is on the lower right.

\[\begin{bmatrix} x - \lambda & 0 \\ -1 & x - \lambda \end{bmatrix}
\]

form $(K - W)$ = rank $(K - N)^2$ and $\epsilon^L$

rank $(K - W)$ = rank $(K - N)^2$ and $\epsilon^L$

$\lambda = \begin{bmatrix} \lambda & \Omega \\ 0 & \lambda \\ 0 & 0 \end{bmatrix}$

the diagonal...

then $\lambda$ = $\lambda$.

Suppose $W$ is diagonalizable.
Let $A$ be a principal ideal domain.

\[ A/p \] is also a principal ideal domain. Let $a$ be a generator of $A/p$.

For every irreducible element $p$ in $A/p$, $a$ cannot be a multiple of $p$.

Thus, $A/p$ is a principal ideal domain.
\[ (x \in \mathbb{C}, \text{ Re}\{x\} = 2) \Rightarrow \exists \lambda \in \mathbb{C} \text{ s.t. } x = \lambda + i \sqrt{3}, \forall \lambda \in \mathbb{C} \}\]

**Theorem:**

Let \( K = \mathbb{C}[x] \) and \( L = \mathbb{C}[x,y] \) be two algebraically closed fields. Then \( K \) is a field.

**Proof:**

Suppose \( \lambda \in \mathbb{C} \) and \( \lambda \neq 0 \). Then \( \frac{1}{\lambda} \in \mathbb{C} \) and \( \lambda + \frac{1}{\lambda} \in \mathbb{C}[x] \) by the fundamental theorem of algebra. Hence, \( K \) is a field.

**Example:**

Let \( \lambda = 2 \) and \( \lambda = 3 \). Then \( \frac{1}{\lambda} = \frac{1}{2} \) and \( \frac{1}{\lambda} = \frac{1}{3} \). Hence, \( \frac{1}{\lambda} \) is also in \( K \) for all \( \lambda \neq 0 \).
So there are no non-trivial elements \( x \in G \). If \( x = 0 \) then \( x = 0 \) and if \( x \neq 0 \), then  
\[
\begin{align*}
\text{dim } \ker(\phi) & = 1 \quad \text{dim } \ker(\phi) \\
\text{dim } \text{image} \phi & = 1 \quad \text{dim } \text{image} \phi
\end{align*}
\]

So \( x = 0 \) and if \( \phi(\alpha) \) is not \( \alpha \), then \( \phi(\alpha) = \beta \) for some \( \beta \). 

So \( \phi(x) = \phi(y) \) if and only if \( x = y \). 

Let \( x = y \) be any element of \( G \). Then \( \phi(x) = \phi(y) \).

\[\text{Proof:} \quad \phi(x) = \phi(y) \iff x = y\]