Let $\mathcal{A}$ be a commutative ring with 1 and let $A, B, C$ be $\mathcal{A}$-modules. Suppose we have a commutative diagram:

\[
\begin{array}{c}
\begin{array}{ccc}
0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\
& & \downarrow & & \downarrow & & \downarrow & & \\
& & 0 & \longrightarrow & D & \longrightarrow & E & \longrightarrow & 0 \\
\end{array}
\end{array}
\]

1. Let $\mathcal{L}(A) = \{a \in A : \forall b \in B, \exists c \in C \text{ s.t. } b = ab + c\}$.

2. Let $\mathcal{L}(A) = \{a \in A : \forall b \in B, \exists c \in C \text{ s.t. } b = ab + c\}$.

3. Find all prime ideals of the ring $\mathcal{L}(A)$. (Hint: Consider the radical of $A$.)

4. Let $\mathcal{L}(A) = \{a \in A : \forall b \in B, \exists c \in C \text{ s.t. } b = ab + c\}$. Suppose $\mathcal{L}(A)$ is independent of $A$ and let $A, B, C$ be $\mathcal{A}$-modules. Suppose we have a commutative diagram:

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5. Let $\mathcal{L}(A) = \{a \in A : \forall b \in B, \exists c \in C \text{ s.t. } b = ab + c\}$. Suppose $\mathcal{L}(A)$ is independent of $A$ and let $A, B, C$ be $\mathcal{A}$-modules. Suppose we have a commutative diagram:

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\]
Let there be a nonzero vector $a \in A$ such that for all $b \in C$.

Suppose that $A$ is odd, and hence of odd dimension.

(c) Suppose that $D$ is odd, and hence of odd dimension.

\[
(b) \chi = \delta 
\]

That is odd (and therefore nonzero).

\[
(b) \chi \sum \chi 
\]

that the complex conjugate of the eigenvalues of \((b) \delta \)

(a) Let $b \in C$. Show that the eigenvalues of \((b) \delta \) are roots of unity and that \((b) \delta \) is
or \( |G/H| = 2 \Rightarrow G \cong C_2 \) or \( G = C_1 \) or \( H \) is the trivial group.

For instance, if \( G \) is a finite group and \( H \) is a normal subgroup of index \( 2 \), then \( G/H \) is isomorphic to \( C_2 \).

\( G/H \) is written as the quotient of \( G \) by the subgroup \( H \). It consists of all the cosets of \( H \) in \( G \).

Clearly, \( H \) is a normal subgroup of \( G \), so \( G = G \triangleleft \) and \( H \) is a subgroup of \( G \).

Let \( a \) act on \( H \) by conjugation. This makes \( H \) a homomorphic image of the
So if we move around and continuously rotate

\[ \theta \leq \pi \text{ rad} \]

\[ \theta \geq -\pi \text{ rad} \]

\[ \theta = 0 \text{ rad} \]

\[ \theta = 2\pi \text{ rad} \]

\[ \theta = \frac{\pi}{2} \text{ rad} \]

\[ \theta = \frac{-\pi}{2} \text{ rad} \]

In the figure, rotate \( \triangle PBC \) so that \( \angle BPC = \theta \). So the image of \( \triangle PBC \)

\[ \angle (\text{OCD}) = \triangle (\text{BEC}) \]

By symmetry, for \( \angle = \pi / 4 \)

\[ \angle (\text{ACD}) = \triangle (\text{BEC}) \]

So we can consider the same case for \( \triangle (\text{ACD}) \)
The solution to \( X^3 - p \) has the corollary of only one root and another.

\[ \text{(a)} \ \exists \alpha \in \mathbb{Q}(\sqrt[3]{p}) \]

For \( \alpha \neq 0 \) then \( \mathbb{Q}(\alpha) \) is not finite and the extension \( \mathbb{Q}(\alpha) = \mathbb{Q}(x) \) is a Galois extension.

\[ \text{Only one of the roots of } \mathbb{Q}(\sqrt[3]{p}) \text{ is a Galois extension.} \]

So, the solution to \( X^3 - p \) is finite. \( \alpha \) is degree 2, so only 3 which imply:

\[ \begin{array}{c}
\alpha_1 = x - \frac{p}{x^2} \\
\alpha_2 = x + \frac{p}{x^2}
\end{array} \]

Thus \( \alpha_1 \) and \( \alpha_2 \) in the fractional exponents \( \mathbb{Q}(x) \) that are finite and hence \( \alpha_1, \alpha_2 \) are finite extensions of \( \mathbb{Q}(\alpha) \).

By the nature of the extension it essential to the roots of the equation.

\[ X^3 - p = x^3 - p = x^3 - \frac{p}{x^2} \]

Which is clearly not finite.

The minimal polynomial of \( \alpha \) is given by:

\[ \text{deg} = 3 \to \alpha \in \mathbb{Q}(\sqrt[3]{p}) \]

Since the degree of \( \alpha \) is finite.

Thus \( \alpha \in \mathbb{Q}(\sqrt[3]{p}) \) is a root of the equation.

\[ \mathbb{Q}(\sqrt[3]{p}) \text{ is a Galois field of the algebraic number} \]

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\[ \int_{0}^{1} \left( x^2 + 3x - 2 \right) \, dx = \frac{1}{3} \]

(\text{The solution of the integral is obtained using integration rules.})

\[ \text{Therefore,} \quad \int_{0}^{1} \left( x^2 + 3x - 2 \right) \, dx = \frac{1}{3} \]

(\text{This is evident from the graph of the function in the interval [0, 1].})

(2) Let \( f(x) = x^2 + 3x - 2 \) and calculate the area under the curve from 0 to 1.

\[ \text{Area} = \int_{0}^{1} \left( x^2 + 3x - 2 \right) \, dx = \frac{1}{3} \]

(\text{By applying the definite integral.})

(3) Let \( \alpha = 1 \) and \( \beta = 2 \), and consider the equation.

\[ \frac{1}{\beta - \alpha} \int_{\alpha}^{\beta} f(x) \, dx = \frac{1}{2} \int_{1}^{2} \left( x^2 + 3x - 2 \right) \, dx = \frac{1}{3} \]

(\text{The average value of the function over the interval [1, 2] is calculated.})

(4) \( \int_{0}^{1} \left( x^2 + 3x - 2 \right) \, dx = \frac{1}{3} \)

(\text{The integral is computed using the Fundamental Theorem of Calculus.})

(5) Let \( f(x) = x^2 + 3x - 2 \) and consider the definite integral.

\[ \int_{0}^{1} \left( x^2 + 3x - 2 \right) \, dx = \frac{1}{3} \]

(\text{The integral is evaluated from 0 to 1.})

(6) Let \( \alpha = 1 \) and \( \beta = 2 \), and consider the equation.

\[ \frac{1}{\beta - \alpha} \int_{\alpha}^{\beta} f(x) \, dx = \frac{1}{2} \int_{1}^{2} \left( x^2 + 3x - 2 \right) \, dx = \frac{1}{3} \]

(\text{The average value of the function over the interval [1, 2] is calculated.})

(7) \( \int_{0}^{1} \left( x^2 + 3x - 2 \right) \, dx = \frac{1}{3} \)

(\text{The integral is computed using the Fundamental Theorem of Calculus.})

(8) Let \( f(x) = x^2 + 3x - 2 \) and consider the definite integral.

\[ \int_{0}^{1} \left( x^2 + 3x - 2 \right) \, dx = \frac{1}{3} \]

(\text{The integral is evaluated from 0 to 1.})