Math 411 — Homework 1— Due June 4, 2010

1. Exercises 1 & 6, Section 10.1

2. Exercise 3, Section 10.2

3. Exercise 4, Section 13.1

4. Instead of defining the usual norm on $\mathbb{R}^n$, we could have instead defined two different norms,
   \[
   \|x\|_\infty = \max_i \{|x_i|\}, \quad \text{and,} \quad \|x\|_1 = \sum_i |x_i|
   \]

   (a) Prove that both of these norms satisfy the triangle inequality.

   (b) Prove that
   \[
   \|x\|_\infty \leq \|x\| \leq \|x\|_1
   \]
   for all $x \in \mathbb{R}^n$.

   (c) We can adapt the definition of an open ball to these new norms, $B_{r,\infty}(x), B_{r,1}(x)$:
   \[
   B_{r,\infty}^\infty(x) = \{v \in \mathbb{R}^n \mid \|x - v\|_\infty < r\}
   \]
   and similarly for $B_{r,1}^1$. Draw a single picture of $B_{1,1}^1(0), B_1(0), \text{and } B_1^\infty(0)$ in $\mathbb{R}^2$.

5. Define a function $f : \mathbb{R} \rightarrow \mathbb{R}$ by
   \[
   f(x) = \begin{cases} x^2 \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0. \end{cases}
   \]

   Prove that $f$ is a differentiable function, but not a continuously differentiable function.