Example G. In a game of bridge, using a standard deck of fifty-two playing cards, each of four players is dealt thirteen cards. Is this an example of a combination or permutation? How many possible bridge hands are there?

combination – The cards were shuffled so that they would not be in any particular order when dealt.

\[
\binom{52}{13} = \frac{52!}{13!(52-13)!} = \frac{52 \times 51 \times 50 \times 49 \times 48 \times 47 \times 46 \times 45 \times 44 \times 43 \times 42 \times 41 \times 40 \times 39!}{13!39!} = 635,013,559,600
\]

Example H. a) Odysseus can name every player on his baseball team’s roster. Is this an example of a combination or permutation? b) As the club’s manager, Persephone decides which player will bat when in the lineup for each game. Is this an example of a combination or permutation? c) If the team roster has 25 members, how many ways can Persephone set up the starting lineup of nine batters?

a) combination – He can name the team in alphabetical order, by position, by size, etc. A specific order is not needed. b) permutation – Order is important. The team will want a different batter to be first up than one who bats in the clean-up position.

\[
c) \ P_{9,25} = \frac{25!}{(25-9)!} = \frac{25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19 \times 18 \times 17 \times 16!}{16!} = 741,354,768,000
\]

Example I-a. One hundred names are put into a hat. Is this an example of a combination or permutation?

combination – The purpose of putting names in a hat is specifically because they are not in any order, and names will be chosen randomly.

Example I-b. One hundred names are put into a hat. Names will be drawn for prizes. The top prize is $500, the next prize is $300, and the third prize is $100. Is this an example of a combination or permutation? How many ways can the prizes be awarded?

permutation – Order matters, especially to the prize winners.

\[
\ P_{3,100} = \frac{100!}{(100-3)!} = \frac{100 \times 99 \times 98 \times 97!}{97!} = 970,200
\]

Example I-c. One hundred names are put into a hat. Names will be drawn for three prizes of $300 each. Is this an example of a combination or permutation? How many ways can the prizes be awarded?

combination – Order does not matter. Prize winners are not worried about whether their names are drawn first, second or third, as long as they’re drawn.

\[
\binom{100}{3} = \frac{100!}{3!(100-3)!} = \frac{100 \times 99 \times 98 \times 97!}{3!97!} = 161,700
\]

Answers to Examples J and K are on the next page.
Example J. For the Powerball lottery, “Every Wednesday and Saturday night, five white balls from 1 to 59 and one red Powerball from 1 to 39 will be drawn. You win a prize by matching some or all of the numbers drawn.” (source: mdlottery.com). Are the choices for a Powerball ticket a combination or a permutation? How many ways can a player pick the numbers for a Powerball ticket?

Both combination and permutation – The five white balls do not have to be drawn in any particular order. However, the single white ball is completely separate from the red ones.

\[
\binom{59}{5} \times \frac{P_{1,39}}{5!(59-5)!} \times 39 = \frac{59! \times 58 \times 57 \times 56 \times 55 \times 54!}{5! \times 3! \times 2! \times 1! \times 54!} \times 39 = 195,249,054
\]

Example K-a. A club is electing its four officers. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can slates of officers be formed?

Permutation – Order matters since each officer has a different job.

\[
P_{4,50} = \frac{50!}{(50-4)!} = \frac{50 \times 49 \times 48 \times 47 \times 16!}{16!} = 5,527,200
\]

Example K-b. A club is forming a committee of four to examine and make suggestions to revise the bylaws. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can the Bylaw Review Committee be formed?

Combination – Order does not matter since the committee members all contribute and jobs have not been assigned.

\[
\binom{50}{4} = \frac{P_{4,50}}{4!} = \frac{50!}{4!(50-4)!} = \frac{50 \times 49 \times 48 \times 47 \times 46!}{4! \times 3! \times 2! \times 1!} = 230,300
\]

Example K-c. A club is forming a committee to plan its annual Equinox Dance. One person will serve as Chair of the committee, and there will be three other members. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can the Equinox Dance Committee be formed?

Both combination and permutation – Order matters in the selection of the Chair, but not in the selection of the other committee members. 921,200

\[
P_{1,50} \times \binom{49}{3} = 50 \times \frac{49!}{3!(49-3)!} = 50 \times \frac{49 \times 48 \times 47 \times 46!}{3! \times 2! \times 1!} = 921,200
\]

Example K-d. A club is forming a committee to plan its annual Holiday Dance. One person will serve as Chair of the committee, another will be Vice-Chair and there will be two other members. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can the Holiday Dance Committee be formed?

Both combination and permutation – Order matters for choosing Chair and Vice-Chair, however, the choice of committee members is a combination. 2,763,600

\[
P_{2,50} \times \binom{48}{2} = 50 \times 49 \times \frac{48!}{2!(48-2)!} = 50 \times 49 \times \frac{48 \times 47 \times 46!}{2! \times 1! \times 46!} = 2,763,600
\]